PROGRAMA DE SEMINÁRIOS EM DESENVOLVIMENTO ECONÔMICO

“A Multisectoral Micro-Macrodynanmic Model”

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Data: 31/03/2005
A Multisectoral Micro-Macrodynamic Model #

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Abstract

This is a simulation model that intends to integrate macro dynamics with structural and sectoral features that shape and modify it. It is an attempt to analyze the macrodynamic effective demand effects of endogenous structural changes in the same setup. Micro-macro interactions result in the model from each level having its own dynamics dependent on the inputs it receives from the others. Each sector is modeled according to neo-Schumpeterian evolutionary microfoundations, with additional unorthodox micro behavioral assumptions. Together with exogenous foreign and government sectors, they are integrated into a multisectoral model. The main features of the model are: (1) simulated sectoral trajectories of a stylized economy derive from endogenous competitive dynamics as well as direct (input-output) and indirect (income, consumption) interactions; (2) sectors are distinguished according to their role in the productive structure and demand categories – consumption, intermediate and capital; (3) no equilibrium is assumed: dynamic interactions among firms’ decisions (based on adaptive expectations) and their effects generate open-ended trajectories. Even though the simulation results presented in this paper only give a very brief idea of the trajectories generated by the model, a general robust result is obtained: the cyclical behavior of the GDP and its main aggregate components.

JEL: O41
Keyword: Multisectoral growth models.

Resumo

Este é um modelo de simulação que pretende integrar a macrodinâmica com aspectos estruturais e setoriais que a conformam e modificam. É uma tentativa de analisar os efeitos macrodinâmicos, em termos de demanda efetiva, de mudanças estruturais endógenas no mesmo arcabouço. Interações micro-macro no modelo decorrem da dinâmica própria de cada nível, ao responder aos comandos recebidos dos demais. Cada setor é modelado com base em microfundamentos neo-Schumpeterianos evolucionários, acrescidos de hipóteses comportamentais micro não-ortodoxas. Juntamente com os setores externo e governamental, eles são integrados em um modelo multissetorial. As características básicas do modelo são: (1) trajetórias setoriais simuladas de uma economia estilizada são derivadas de uma dinâmica competitiva endógena, bem como de suas interações diretas (insumo-produto) e indiretas (renda, consumo); (2) os setores se distinguem por sua inserção na estrutura produtiva e nas categorias de demanda – consumo, intermediários e capital; (3) não há nenhum pressuposto de equilíbrio: interações dinâmicas entre decisões das firmas (baseadas em expectativas adaptativas) e seus efeitos geram trajetórias em aberto. Embora

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# This paper is part of an integrated research project supported by CNPq (Brazil’s National Research Council). Ana Cristina Reif Visconti, a PhD student also working in the project, is co-responsible for all macro sections. The sectoral part of the model draws heavily on Possas, Koblitz et al. (2001).
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os resultados de simulação apresentados neste artigo dêem apenas uma breve ideia das trajetórias geradas pelo modelo, um resultado geral robusto é obtido: o comportamento cíclico do PIB e de seus principais componentes agregados.

Palavra-chave: Modelos de crescimento multissetoriais.
1. Introduction and theoretical assumptions

This paper presents a micro-macro multisectoral evolutionary simulation model that combines neo-Schumpeterian evolutionary microfoundations with some post-Keynesian and Kaleckian assumptions and exhibits some preliminary results of simulation runs made on selected issues. The main objective of the model is to put together analytical elements that may be useful to investigate dynamic properties of capitalist economies which depend mainly on micro-macro relations, with a special regard to the analysis of economic development. It is our belief that very important complementary insights and results can be drawn from combining these approaches¹. As a starting point, both theoretical fields share the rejection of two neoclassical foundations: (i) substantive rationality; and (ii) equilibrium of agents and markets.

Concerning rational decision processes, both fields assume (or at least are compatible with) bounded and procedural rationality, as developed by Simon (1983), through which instrumental rationality may be reconciled with hard uncertainty (in the sense of Knight and Keynes). As is well known, the latter is supposed to be a feature of economic environments where irreducible information and competence gaps (in both cognitive and computational senses) can emerge. In such context, rationality involves “satisficing” (apud Simon) kinds of sub-optimal solutions that may lead to strategies based on routines and conventions (Heiner, 1983; Nelson and Winter, 1982).

As for the rejection of the notion of equilibrium, neo-Schumpeterian and post-Keynesian approaches usually share the view that disequilibria and coordination failures are normal in a market economy. But this amounts to assuming, in more formal terms, a nonergodic and nonstationary economic environment, in which rational agents can make systematic forecasting mistakes, as opposed to the rational expectation hypothesis (Vercelli, 1991, p. 154-5). In particular, for the traditional neo-Keynesian economic growth and fluctuations theories up to the 60’s, disequilibrium was essential in explaining capitalist economic dynamics, either in the more conventional interpretation as causing the propagation of fluctuations around a trend of moving equilibrium, or even when such trend is seen as irreducible to an equilibrium in any intelligible sense (Kalecki, 1954; Possas, 1983, 1999).

Both kinds of theories also admit that capitalist economies show regularities that may reduce uncertainty (without eliminating it) and allow long run decisions to be made, thus mitigating the effects of potential instability (Vercelli, 1991, ch. 5; Possas, 1993). But these regularities do not prevent capitalist economies from exhibiting nonlinearities originated from cumulative decisions and their structural effects (technological paths with technical progress and learning, synergies, etc.), which may cause strong structural instability. Technical progress and corresponding technological trajectories (Dosi 1982, 84) is probably the main dynamic process causing such effects in the long run, and not just through their direct innovative impacts. At the same time, they usually increase dependence on existing assets, acting as a source of increasing returns and sunk costs that create path dependence and lock-in effects in long run paths. Stable institutions may induce similar effects, although more complex and less studied by economists.

¹ For recent attempts see Verspagen (2002) and a survey by Llerena, Llorentz (2003).
Main contributions from the neo-Schumpeterian evolutionary approach (Nelson and Winter, 1982; Dosi, 1984) are incorporated by including explicitly in the model its two theoretical cornerstones: (i) behavior diversity among agents, endogenously generated through search of opportunities to innovate; and (ii) a selection of firms, strategies and/or technologies basically through market competition, for which no reference to equilibrium is needed.

Feedback between strategies and selection through the market (or other institutions) entails an endogenous industrial dynamics. Industrial structure and performance emerge from this interaction across patterns of technological change that may shape technological trajectories (Dosi 1982, 84). A successful innovation allows a firm to reach competitive advantages and fetch larger profits and/or market shares, thus raising asymmetries not only in performance variables, but also in market structure (Dosi 1984, 88). Iterative processes based on change in parameters and/or in expectations by firms give place to open ended dynamic paths without any equilibrium trend, where not even self-organizing order or regularities are necessarily expected to be found.

In spite of being largely unpredictable, we believe that such long run trajectories can be successfully studied through simulations based on specific hypotheses concerning parameters and initial conditions. In fact, the performance of simulation exercises to investigate the basic dynamic properties of economic market processes of change has become a typical feature of the evolutionary neo-Schumpeterian research program, since one cannot expect analytical solutions usually to emerge for such complex system modelling - except under seriously restrictive assumptions, which can bring them close to irrelevance.

A surge of neo-Schumpeterian evolutionary models trying to analyze sectoral dynamics along these lines emerged in the last two decades². The path breaking work of Nelson and Winter (1982) paved the way for a family of models of Schumpeterian competition. In the second round of these models, by the beginning of the 90’s, there was a split within evolutionary/neo-Schumpeterian models: (1) “microdynamic models”, related to industrial trajectories with technological change; and (2) “endogenous growth models”, more related to macroeconomic issues, clearly as a counterpoint to neoclassical endogenous growth models. But in spite of the effort to include important elements left out by neoclassical growth models, such models still present a serious flaw – there is no underlying economic structure, essential to any macroeconomic analysis, including the sectoral interrelation among consumption, investment and intermediate goods. The transition from micro to macro levels is done without macro-sectoral mediations between the firms and the economy as a whole, either through input-output relations or through income generation and final demand. In a few words, it means that these models have no macroeconomic level of analysis. The importance of such sectoral interrelations, however, was stressed by some evolutionary authors: “the structure of input-output, as well as the untraded technological interdependencies of each economy, can be regarded as a huge feedback machine that amplifies, transforms or smoothes technological and demand impulses generated in any part of the economy, transmitting them to the rest of the system in ways which are both sector-specific and country- (or region-) specific” (Dosi, Pavitt and Soete, 1990, p. 108).

2. Main features of the model

**Multisectoral model.** Input-output (sector x sector) matrices are employed together with other expenditure matrices – consumption (sector x personal income class) and incremental capital/output (sector x sector) - to endogenize main components of final demand. At the income generation side, matrices of income appropriation within classes (functional income class x sector) and of personal income appropriation (personal income class x functional income class) are defined. Other components of final demand, as exports and government expenses, are left exogenous.

**Dynamic model.** It generates trajectories in discrete time (periods). Since causality is based on decisions to produce and to spend (effective demand), no equilibrium positions are ever required. The use of given matrix coefficients do not prevent dynamic modeling, because it only requires such coefficients being fixed during each simulation period, while it is possible to change them between periods according to some established rules. Since periods are defined as a time lapse between consecutive decisions (production, investment, consumption), this assumption poses no consistency problem, given that decisions in any case could only be revised by the end of each period.

**Firms are the basic units.** Each firm belongs only to one sector. Structural changes in each sector are endogenously dependent on firms’ behavior, especially as a result of technological and strategic diversity. Conversely, firms try to adapt to market conditions through feedback mechanisms. Some basic features are: (i) prices are decided by firms according to expected markups, subject to endogenous change due to strategic market concerns; (ii) effective demand causality in production decisions and sales (i.e. absence of market supply and demand equilibrium) involves distinguishing between output and sales, as well as putting emphasis on short period expectations concerning sales, assumed to be endogenous (extrapolative); and (iii) investment decisions follow basically the same rule, but allowing for an important autonomous component related to technical progress, and imposing financial debt constraints. These mechanisms can be more deeply explored within the multisectoral structure, which allows treating as exogenous some important elements which otherwise could only be fixed exogenously.

More specifically, firms’ strategies and decisions can be divided in three subsystems:

(i) production, prices and profits;

(ii) investment;

(iii) technological search.

In the first one, basic “effective demand” elements are drawn from Possas (1983, 84): production decisions are based on expected sales for the production period, extrapolated from the average of some previous periods. As to prices, the present model assumes each sector to be an oligopoly with some degree of price competition as well as of product differentiation, following a version of Kalecki’s price model (1954, ch. 1), in which actual price is a weighted average of the price corresponding to the expected markup and the

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3 Based on the sectoral evolutionary model in Possas, Koblitiz, et al. (2001). But while in that case the only sector was modelled as a science based one, in the present case sectors are widely distributed across Pavitt’s (1984) taxonomy.

4 The exceptions are firms in the intermediate and capital goods sectors, which produce according to their current orders.
industry average price, but subject to change according to a feedback from the firm’s competitive performance.

Investment decision rules on new capacity are also drawn from Possas (1983, 84), based on extrapolated expected sales from some previous (investment) periods but limited by a debt constraint following Kalecki’s principle of increasing risk (1954, ch. 8). Wider financial features are included in these decisions to capture the influence of assets and liabilities structure of the firm, represented by debt/equity ratio, retained profits and liquidity demand.

Lastly, technological search combines different approaches: both innovative and imitative searches follow a stochastic process as in Nelson and Winter (1982); and a learning process is also included drawing on the vintage model by Silverberg, Dosi, and Orsenigo (1988), from which a payback period criterion for equipment replacement decisions is also applied.

**Interactions at the sectoral level.** Demand for each sector is in part determined endogenously by firms and household decisions to spend and in part exogenously by exports and government expenditure, and is divided among firms by a “replicator” dynamic equation. Production and investment decisions by each firm determine, respectively, the demand for intermediate and for capital goods, and household decisions determine demand for consumption goods.

Consumption is a function of the average income of each income class, assumed to be linear and to have higher lags and lower propensity to consume for higher income classes. The income flowing to each class is calculated as a proportion of the total amount of wages and distributed profits. The distribution of the value added between wages and profits in each sector is a function of the average markup and the unit wage, which can be assumed to change across periods.

**Exogenous blocks.** In addition to the above endogenous core, the model also involves three partially exogenous blocks or “sectors” treated separately: foreign sector (trade and capital flows); government (public expenditure, taxes and economic policy); and a financial sector (debts, capital investment and interest rates). This treatment allows an easier setting of specific simulation assumptions concerning strategic areas for macrodynamics and, in particular, for economic development. A multicountry model could be a future extension of the model, therefore endogenizing the foreign sector.

3. The model

In its general specification, the model defines an economy with $m$ income classes $(1,\ldots,h,\ldots,m)$, $p$ sectors $(1,\ldots,z,\ldots,p)$, at least three, $n$ firms in each sector $(1,\ldots,i,\ldots,n)$, each one initially containing $l$ capital goods $(1,\ldots,j,\ldots,l)$. Following one of the present trends in neo-Schumpeterian evolutionary modelling, the model was built and the simulations were run on the software Laboratory for Simulation Development (LSD), details of which can be found in Valente (1999).

*Block 1: Production*

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5 Although the complete model involves all the equations described below, not all will be used in every simulation run.
1A. Planned production

\[ x_{it}^* = x_{it}'(1 + \sigma) - x_{it-1}' \]

(1),

subject to \( 0 \leq x_{it}^* \leq \bar{x}_{it} \),

where \( \sigma \) is exogenously fixed.\(^6\) The production decision \( x_{it}^* \) at the beginning of period \( t \) is aimed at two goals: (i) to meet the expected demand for sales \( x_{it}^e \) at the end of the production period beginning at \( t \); and (ii) to keep the stock \( x_{it}^s \) at a safe level to cope with unexpected demand fluctuations, which is assumed as a fixed proportion of sales, \( \sigma \).\(^7\)

Production is limited by existing productive capacity \( \bar{x}_{it} \), measured in production units. Since different equipment units have different productivities, we assume the most efficient ones to be used first.

In the capital good sectors each firm produces based on current orders, \( e_{i,t} \), its planned production being equal to the previous orders,\(^8\) subject to the same capacity constraint:

\[ k x_{i,j}^* = k e_{i,t} \]

(1'), subject to \( 0 \leq x_{i,j}^* \leq \bar{x}_{i,j} \).

1B. Expected sales

Firm’s sales expectations follow an extrapolative rule\(^9\) from past effective orders \( e_i \):

\[ x_{i,t}^e = e_{i,t-1} + \gamma (e_{i,t-1} - e_{i,t-2}) \]

(2),

with \( \gamma \) exogenously fixed.\(^10\)

Block 2: Total sectoral demand and firms’ demand

2A. Total sectoral demand

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\(^6\) In the simulations, \( \sigma = 0.1 \). This and other fixed parameters were defined by “educated guesses” and are assumed to be the same across firms, except when otherwise stated.

\(^7\) The production decision follows Metzler (1941); see Gandolfo [(1985), p.90], and Possas (1983).

\(^8\) Since planned production is the basis on which every sector decides its demand for intermediate goods, planned production of intermediate goods also has to be based on past orders, and effective production on current orders.

\(^9\) Gandolfo (1985), p. 95, suggests that this equation was originally proposed by Goodwin (1947). For a taxonomy of simple expectation formation rules see Williamson (1989), p.214.

\(^10\) In the simulations, \( \gamma = 0.5 \).
Total demand is specified differently for each sector, so it will be presented separately. Note that in the case of both capital and intermediate goods, total demand will determine planned production.

(i) Intermediate goods sectors

The effective production of each firm in these sectors is constrained both by its productive capacity and by the stock of intermediate goods it holds. The amount of inputs produced by sector $z$ necessary to meet the planned production of firm $i$ is given by fixed technical coefficients:

$$x_{z,i,t} = x_{z,i}^* \left( a'_{z,i,t} + a''_{z,i,t} \right) \tag{3},$$

or, in matrix form:

$$X_i^m = (A_i + A_m) \tilde{x}_i^* \tag{3'},$$

where $X_i^m$ is the ($p \times pn$) matrix of required inputs; $A_i$ and $A_m$ are the input-output ($p \times pn$) matrices of domestic, $a_{z,i,t}$, and imported, $a_{m,z,i,t}$, technical coefficients; and $\tilde{x}_i^*$ is the diagonal matrix ($p \times pn$) of planned production of each firm $i$, $x_{i,t}^*$.

If the required amount of each intermediate good is available, the quantities spent will be those indicated on the columns of the matrix $X_i^m$; otherwise, only a proportion $\rho_{i,t}$ of these quantities will be used for each firm $i$. Therefore, the total demand for domestic intermediate goods at t+1 is determined by each firm’s planned production, $x_{i,t+1}$, as calculated by an extrapolation rule, deducting its previous unused stock of intermediate goods:

$$i' e_i = \left( A_i \tilde{x}_{i,t+1} - X_{i,t}^{i\tau} \right) \bar{u} \tag{4},$$

with

$$X_{i,t+1}^{i\tau} = x_{i,t}^* \left( 1 + \gamma_i \lambda_{i,t} \right) \tag{5},$$

where $i' e_i$ is the vector ($p \times 1$) of domestic demand for intermediate goods sectors, $X_{i,t}^{i\tau}$ is the ($p \times pn$) matrix of the unused stock of intermediate goods and $\bar{u}$ is a column unitary vector.

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11 This proportion is equal to lowest ratio for the firm $j$ of the available intermediate goods of sector $i$, $x_{i,j,t}^{i\tau}$, and the amount required, $x_{i,j,t}^{i\tau} \cdot \rho_{j,t} = \min \left( \frac{x_{i,j,t}^{i\tau}}{x_{i,j,t}^{i\tau}}, \frac{x_{i,t}^{i\tau}}{x_{i,t}^{i\tau}} \right)$.

12 In the case of intermediate goods sectors, since they produce based on these orders, the remaining stock of inputs is not deducted since it is not yet determined.

13 The columns corresponding to intermediate sectors in the matrix $X_{i,t}^{i\tau}$ are zeros.
(ii) Consumption goods sectors

The domestic demand for consumption goods is determined by households’ income, according to their income class, and government expenses. The consumption of each class is assumed to be a linear function, with increasing lags14 and decreasing marginal propensity to consume, of the average real income of each class, \( \bar{y}_h \), plus a fixed autonomous consumption:

\[
\ell^t e_i^t = C\bar{y}^t + C^d \bar{u} + c_i^g
\]

(6),

where \( C \) and \( C^d \) are \((p \times m)\) matrices of the marginal propensity to consume and autonomous consumption, respectively, \( \bar{y}^t \) is the vector \((m \times 1)\) of the average real income of each class; and \( c_i^g \) is the vector \((p \times 1)\) of the government consumption measured in output units of each sector.

(iii) Capital goods sectors

The investment decisions on fixed capital (for expansion and replacement of productive capacity) have two major components: expected sales and the financial risk of increasing indebtedness15. Firms take these decisions at every investment period, which by assumption equals six production periods16, corresponding to the time lag needed to produce, install, and start operating the new equipments. In order to be implemented, such decisions must be financially feasible, i.e., the firm must be capable of paying for the new capital goods either with its own and/or with borrowed resources, as it will be explained bellow. A proportion17 of the aggregate demand for capital goods is for imported ones; thus, the orders received by the domestic capital good sector are determined deducting imports and adding government investments:

\[
^k e_i^t = ^k X_i \left( \bar{u} - ^k m \right) + \hat{p}_{t-1}^i I_t^g
\]

(7),

where \(^k e_i^t\) is the \((p \times 1)\) vector of orders received by the domestic sectors; \(^k X_i\) is the \((p \times pn)\) matrix formed by capital goods total demand vectors; \(^k x_i,^k m\) is the \((pn \times 1)\) vector of import coefficients for capital goods; \(^k I_t\) is the \((p \times 1)\) vector of government investment expenditure and \( \hat{p}_{t-1}^i \) is the \((p \times p)\) diagonal matrix of the inverse of last period average prices18.

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14 The simulations assume \(m=4\). For class A, a 4 period lag is assumed; for class B, 3; for class C, 2; and, for class D, 1.
15 That is, the debt/capital (or debt/equity) ratio.
16 This version of the model assumes the same investment period for all sectors, although decisions are not simultaneous.
17 The ratio is sector specific; however, in the standard simulations it is set to 5% for all sectors.
18 Given technical indivisibilities, the effective level of government investment is determined by the integer component.
(iv) Effective orders

Finally, effective orders for each sector are determined by the sum of domestic and foreign demand.\(^{19}\)

\[ e_i = c^i e^i_t + k^i e^i_t + l^i e^i_t + e^m_i \]

(8), where \(e^m\) is the \((p \times 1)\) vector of each sector’s import orders, defined below at the foreign sector block.

2B. Firms’ demand: replicator dynamic equation and competitiveness

The discrete formulation presented here was developed by Kwasnicki and Kwasnicka\(^{(1996)}\), based on Silverberg’s\(^{(1987)}\) adaptation to firms’ competition of the original equation developed by Fisher. The only difference here is an adjustment parameter \(\mu\)^{20}.

\[
S_{it} = S_{it-1} - \left[1 + \mu \left( \frac{E_{it}}{\bar{E}_t} - 1 \right) \right] \quad \text{or} \quad \frac{S_{it} - S_{it-1}}{S_{it-1}} = \mu \left( \frac{E_{it}}{\bar{E}_t} - 1 \right)
\]

(9),

such that \(0 \leq \mu \leq 1\) and \(\bar{E}_t = \sum_{i=1}^{n} E_{it}. S_{it-1} - 1\),

where \(E_i\) is a competitiveness index for firm \(i\), based on price and delivery delay.

In the model, each firm’s competitiveness is defined as:

\[
E_{it} = \frac{1}{p_{it} \cdot d_{it} \cdot \epsilon_{it}}
\]

(10),

where \(p_i\) is the price and \(dd_i\) is the delivery delay for firm \(i\), and \(\epsilon_1\) and \(\epsilon_2\) are respectively the firm’s competitiveness elasticities relative to price and to delivery delay.

The above definition is consistent with Silverberg’s\(^{(1987), p. 121}\) comment that relative, not absolute, price differences may drive a customer away from a seller to another. Silverberg introduces the price logarithm in his definition to keep up with that observation. In our model this device is unnecessary since a different replicator equation is used: while in Silverberg’s model market share depends on the absolute difference between individual and average competitiveness\(^{21}\), here it is the ratio between individual and average competitiveness that fulfills this role. Therefore, in defining competitiveness as a function of price, the market share for each firm will be determined by relative prices.

Block 3: Firms’ orders, actual production and sales

\(^{19}\) In general the sectors are specialized, which means that only one of the three domestic components is positive.

\(^{20}\) This parameter is specific to each sector. A discussion on the consistency of this equation is presented in Possas, Koblitz\ et al.\ (2001).

\(^{21}\) The equation used by Silverberg is \(\frac{df_i}{dt} = A(E_i - \bar{E}) f_i\), where \(f_i\) is the market share of firm \(i\).
3A. Firms’ orders

Effective orders received by a firm depend on total sector demand, $e_i$, and on the firm’s market share $s_{it}$, determined by the replicator dynamic equation, under the effect of firm’s competitiveness:

$$e_{ij} = s_{ij}e_i$$

(11).

However, orders actually received by an individual firm might differ from this level if some firm is unable to meet its orders and other firms in the same sector have excess supply. The resulting excess demand is divided among remaining firms on the basis of their market share ranking.

3B. Actual production and sales

Actual production is planned production subject to the intermediate goods constraint:

$$x_t = \hat{x}_i \times \rho_t$$

(12),

where $x_t$ is the $(pn \times 1)$ vector of the effective production, $\rho_t$ the $(pn \times 1)$ vector of elements $\rho_{it}$ and $\hat{x}_i$ was defined before.

In the case of firms belonging to an intermediate sector, actual production is not based on planned production, but on actual orders, as in capital goods sector. In this case, however, the aim is not only to meet the domestic and foreign current orders, but also to keep the stock $x_{it}^s$ at an acceptable level, in face of unexpected demand fluctuations (as in the consumption goods sectors):

$$x_{it}^s = e_{ij} \times (1 + \sigma_i) - x_{it}^*$$

(13),

subject to $0 \leq x_{it}^* \leq x_{it-1}$.

Actual sales $x_t^v$ are determined by the effective orders, which may or may not correspond to the expectations that previously defined the level of production. This interaction between sales and production over time creates a mechanism of dynamic induction over the subsequent production decisions, via changes on the expected behavior of future sales. Obviously sales level cannot exceed the production level plus existing stocks:

$$x_t^v = x_t^s$$

(14),

subject to $0 \leq x_t^v \leq x_t + x_{t-1}^s$.

Finished goods inventories are, by definition:

$$x_t^s = x_{t-1}^s + x_t - x_t^v$$

(15).
The stock of intermediate goods available for next period production is given by the amount not used in the current period, $x_{i,j,t}$, plus the amounts purchased domestically and imported:

$$x_{i,j,t} = x_{i,j,t}^{\text{in}} + e_{i,j,t}^{\phi} + x_{m}^{i,j}$$

(16), where $\phi_{i}$ is the proportion of the orders that sector $i$ could meet; and $x_{m}^{i,j}$ are sector $j$ imports of sector $i$ goods. If effective orders (in the first round) received by a firm exceed the sum of its planned production and available stocks, the firm will incur in a delivery delay, which will have a negative impact on its competitiveness in next period. The delivery delay figure compares effective orders with effective sales:

$$dd_{i,t} = \frac{e_{i,t}}{x_{i,t}^{\phi}}$$

(17).

**Block 4: Prices and costs**

**4A. Price decisions**

The price equation used here, as shown elsewhere, is a discrete version of Silverberg’s, consistent with the version specified before for the replicator equation and it is also identical to the price equation used by Kalecki (1954, ch. 1) in his analysis of the “degree of monopoly” of a firm under imperfect competition:

$$p_{i,t} = \theta p_{i,t}^{d} + (1 - \theta) \bar{p}_{i,t-1}$$

(18), or

$$k_{i,t} = \theta k_{i,t}^{d} + (1 - \theta) \frac{\bar{p}_{i,t-1}}{u_{i,t}}$$

(18a), where $p_{i,t}^{d} = k_{i,t}^{d} u_{i}$ is the firm’s desired price for each period, i.e., the price that results from applying the desired markup $k_{i,t}^{d}$ over the unit variable cost $u_{i}$; and $k_{i}$ is the effective markup corresponding to the effective price $p_{i}$.

As mentioned above, the latter equation is exactly the one used by Kalecki (1954, ch. 1). Both Kalecki and Silverberg look at their equations as simple extensions of the so-called “full cost principle” to oligopolistic conditions, where it is impossible for firms to ignore each other’s prices. Alternatively, it can be understood as one of the determinants of markup in oligopoly: as a sort of compromise between the desired markup by a firm (or its long run strategic markup) and current competitive conditions. While low cost firms enjoy the advantage of making additional profits in the short run, in excess of what would result from applying the strategic markup, high cost firms sacrifice their desired markup for keeping their market share (Silverberg, 1987, p.130).

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22 Some degree of substitutability between domestic and imported goods of the same sector is assumed.

23 Possas, Koblitz et al. (2001).
Another behavioral implication of this equation is that, since the average price is weighted by market shares, larger firms will have a greater influence on average market price, thus playing a kind of price leadership, while small firms can substantially reduce prices without producing a large impact on market price (to an amount required, for instance, to start a price war).

Each firm’s unit variable cost in a given period, $u_{it}$, is the sum of unit input costs $m_i$, assumed constant as a function of quantity produced, and unit labor cost, which depends on the nominal wage rate $w_i$ (assumed constant over time and with the amount produced) and on the firm’s average productivity, $\bar{\pi}_i$ (see below):

$$u_{it} = m_i + \frac{w_i}{\bar{\pi}_{it}}$$

(19),

with $w$ given as initial condition; and

$$m_i = A_i^p p_{it-1} + e \pi_i^m p_{it}^m$$

(20),

where $p_i$ and $p_{it}^m$ are $(n \times 1)$ domestic (sector average) and foreign price vectors, respectively, and $e \pi_i$ is the exchange rate$^{24}$.

4B. Technological routines and productivity

Average labor productivity for each firm varies over time as a function of (i) the investment on fixed capital and the degree of productive capacity utilization; (ii) the R&D strategy adopted; and (iii) the efficiency of the learning-by-doing process.

Fixed capital stock at any period is heterogeneous, composed of equipments requiring different labor productivity to operate, so that the firm’s average productivity depends on which capital goods are being used and on their degree of utilization. Each equipment’s productivity at a given period, on its turn, results from the combination of the outcome of the firm’s technological search at the moment it was ordered (more details later) and of the improvements obtained while using it, associated with the adjustment processes that must be done, along with the mentioned learning by doing process$^{25}$. These advantages$^{26}$, however, are balanced by two other factors also present in the model: (i) the learning-by-doing effects were realistically assumed to be limited; and (ii) they are specific to each equipment/technology, so that when the latter is replaced, the firm enters into a different “learning curve”$^{27}$.

(a) Productivity of each equipment/technology:

$^{24}$ For simplification, the exchange rate is being kept constant.

$^{25}$ Scherer and Ross (1990), ch. 4, pp.97-98. Specific product economies of scale are those associated with the amount produced and sold of only one product.

$^{26}$ The difference of these advantages among sectors will be taken into account. According to Scherer and Ross, op. cit., “...some of the product lines in which learning-by-doing is most important (such as semiconductors, aircraft, and computers) are also characterized by rapid technological obsolescence of product designs. The development of a completely new design often permits an initially handicapped producer to jump to a new learning curve in a position of equality or even superiority” (p.372).

$^{27}$ More details in Scherer and Ross (1990) and Possas, Koblitz et al. (2001).
\[ \pi_{i,j,t} = \pi_{i,j,t}^0 h_{i,j,t} \]

(21),

where \( \pi_{i,j,t}^0 \) (initial productivity of equipment \( j \) of firm \( i \)) is determined in Block 6 below, and \( h_{i,j,t} \) is defined afterwards.

(b) Learning effect (learning-by-doing):

\[ h_{i,j,t} = 1 + z(1 - \exp(-\tau \sum_i x_{i,j,t}^*)) \]

(22).

The parameters \( z \) and \( \tau \) of this equation represent respectively the growth rate of the equipment’s initial productivity that can be reached through learning-by-doing, and the speed with which it can reach this level\(^{28}\).

**Block 5: Investment decisions and financial constraint**

Investment decisions determine both the firm’s average productivity and the extent to which it can grow in the long run. The model considers two components – apart from technological improvements - of an investment decision: capacity expansion and capacity replacement. The latter can be explained either by a thorough physical depreciation or by technological obsolescence, or both. In order to be implemented, such decisions must be financially feasible, i.e., the firm must be capable of paying for the new capital goods either with its own and/or with borrowed resources, subject to a given precautional demand for liquid assets and to an upper indebtedness bound. These financial variables act as a constraint to the firm’s desired investment. This Kaleckian (and partly Keynesian) provision is a clear improvement upon the traditional “accelerator” mechanism. The main differences from our model as compared to Kalecki’s (1954, ch. 9) are: (i) the introduction of a financial constraint, instead of adding it as a continuous variable, on the investment equation; and (ii) the “accelerator” component itself, adapted to cope with the necessary adjustments of the degree of capacity utilization together with the observed growth projection (Possas, 1987)\(^{29}\).

At the same time, detailed descriptions of the investment decisions made by behaviorist economists\(^{30}\) seem to be largely consistent with investment decision routines and with their relation with financial variables, proposed by the present model\(^{31}\).

**5A. Investment decisions**

Investment decisions are taken at the end of each investment period (time interval between consecutive investment decisions), which is assumed to comprise six production periods each.

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\(^{28}\) Both parameters are given as initial conditions.

\(^{29}\) Possas (1987) made a detailed critical discussion on investment determinants in Kalecki’s model of 1954, as well as on the accelerator and for the original formulation of the equation used here.

\(^{30}\) See Cyert et alii (1979) and Bromiley (1986), quoted in Possas, Koblitz et al. (2001).

\(^{31}\) For instance, concerning the role of financial variables as constraints to desired investment, see Cyert et alii (1979), op. cit., in Cyert and De Groot (1987), p.134; apud Possas, Koblitz et al. (2001).
Decision making starts with a forecast of average sales for the next production periods \([t+6; t+12]\) when the new capacity, resulting from current investment, will be operative – by definition, its “construction period”, also assumed to be equal to the investment period. This forecast is a simple extrapolation of average sales of the corresponding previous investment periods. Expected sales for the next investment period \(x_{T+1}\) are therefore:

\[
x_{T+1} = e_T + \gamma(e_T - e_{T-1})
\]

(23),

where \(e_T\) = average orders in the current investment period\(^3\), by the end of which decision is being made. Assuming that the same absolute change in demand will repeat itself in the following period,

\[
x_{T+2} = x_{T+1} + \gamma(e_T - e_{T-1}) = e_T + 2\gamma(e_T - e_{T-1})
\]

(24).

In order to determine the desired productive capacity one needs to know the production level that is expected to be necessary, which, as usual, has not only to meet expected sales, but also a given stock level. The latter was established before as a fraction \(\sigma\) of expected sales. As a safety margin for possible forecast errors and unforeseen demand fluctuations, the above result is taken as a fraction \(\alpha\) of planned capacity. Finally, to obtain the variation in planned capacity which will justify investment, one needs only to subtract the existing capacity. Thus, desired productive capacity is determined by\(^4\):

\[
\Delta x_{i,T} = \frac{(1 + \sigma_i)}{\alpha_i} x_{i,T+2} - (1 - \delta) \bar{x}_{i,T}
\]

(25),

where \(\Delta x_{i,T}\) is the desired increase in productive capacity \(\bar{x}_{i,T}\), while \(\bar{x}_{i,T}\) is the planned capacity, \(\alpha\) as defined above (given as an initial condition) and \(\delta\) the depreciation rate.

Now, the value of desired gross investment in fixed capital is obtained by multiplying the desired increase in capacity, in addition to the amount of physical capital replacement, by the price of capital goods:

\[
I_{i,T} = p_{k,i}(\Delta x_{i,T} + \bar{x}_{i,T})
\]

(26).

5B. Financial constraint to investment\(^5\)

Total financial funds available for investment, \(F\), are given by:

\[
F_{i,T} = F_{i,T} + F_{i,T} - A_{i,T}
\]

(27).

\(F\) is the amount of the internal funds (or cash flow), resulting from deducting taxes and distributed profits from net profits (which defines retained profits \(P^R\), and adding

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\(3\) In this block of equations, subscript \(T\) refers to the investment period, where \(T=t/6\).

\(4\) The average orders over the last 6 production periods.

\(5\) This sets the maximum level of required capacity, in order to meet product demand and stock replacement needs.

\(5\) The financial constraint used in this model was largely inspired by Wood (1975), and also employed in Possas (1984). For more details see Possas, Koblitz et al. (2001).
depreciation funds. $F^{X}$ are the external funds that a firm may borrow up to a given acceptable rate $g$ of debt on total capital, exogenously assumed in the simulations as fixed and equal among firms. These funds will only be used when internal ones are insufficient to fund the amount of desired investment\textsuperscript{36}. The firm is also supposed to keep a given amount of liquid resources as a means of avoiding short run borrowing due to sales forecasting errors. $A^*_t$ is the amount of desired additional liquid funds as a proportion $\phi$ of the already existing liquid capital stock\textsuperscript{37}.

When the firm’s rate of debt on capital exceeds a given risk threshold, which is much above the acceptable level $g$ (in the simulations, 90%), the model assumes it has failed and it will thus be eliminated from the market.

Now, a financial constraint on the desired value of total investment can be applied, depending on a number of factors, as shown in the above equation. To simplify matters, we can identify the following main alternatives:

(i) The amount of total financial funds available for investment is negative ($F_{i,t} < 0$).

This situation may result from high losses, high indebtedness, liquidity squeeze or a combination thereof. Generally it will take place if $F^I$ is small or negative; and the firm’s reaction will depend on its stock of liquid assets. Should it be insufficient to cover (the negative value of) $F_{i,t}$, the firm will use it up to reduce the debt or to reduce the impact of $F^I$ in case of loss. Otherwise, liquid funds will be reduced in the amount of $F_{i,t}$. In any case the investment in new capacity, as well as that eventually designed to technological updating, will amount to zero.

(ii) The amount of total financial funds available for investment is positive or null ($F_{i,t} \geq 0$).

Two situations may happen:

a) available funds are less than or equal to desired investment. In this case, the firm will invest the amount available, taking into account the technical indivisibility of investment (the minimum unit of capacity was set as 10 production units in the model). Effective financial flows (external funds plus liquid assets investment) will equal the values that entered in the calculation of $F_{i,t}$, and possible residues due to fixed capital indivisibility will be used to increase liquid assets;

b) available funds are greater than desired investment. The firm will be able to invest the desired amount, and the remaining surplus will be destined (when required) to technological updating of the equipment. If these funds are completely used up, effective financial flows will equal their initial values.

Lastly, desired investment in technological updating of the equipment will be determined by a common payback rule for each unit of capital equipment, starting with those of lower productivity:

\textsuperscript{36} $F^{X}$ may be positive – when external resources may be added to internal ones to finance investment without exceeding an acceptable level for the rate of debt on capital; or negative, otherwise – in which case part of the internal funds should be used to reduce the debt. This debt adjustment is made stepwise to reflect some tolerance of the firm to exceed its debt limit so as to avoid sacrificing the whole desired investment.

\textsuperscript{37} $A^*_t$ may also be positive or negative, whether or not an increase in liquid resources is needed.
\[
\frac{p_{k,j}x_{i,j,t}}{w\left(1 - \frac{1}{\pi_{i,j,t}^b} \right)} \leq b
\]

(28),

where \( \pi_{i,j,t} \) is the productivity of equipment \( j \) of firm \( i \) and \( b \) the payback period.

If after this stage there still remains some liquid surplus, its destination will depend on its amount. The firm is supposed to use its own funds in the first place, so if the surplus is greater than or equal to the external available funds, the firm will not run into new debt and any remaining surplus will be kept as liquid assets. Conversely, should the external funds be greater the same rule would apply: the firm will only use external funds to the amount strictly necessary to invest.

**Block 6: Frontier shift and technological search**

Technological search by any firm is accomplished through process R&D. The assumption made here is that the industrial sector being modeled introduces technical change basically embodied in the equipment ordered, but at the same time internal R&D is assumed to be crucial for design and technical improvement of the equipment, through learning-by-doing (in K. Pavitt’s (1984) taxonomy, it would correspond closer to “scale intensive”, with some elements of “science based”, sectors). The innovation and diffusion (imitation) processes follow closely those 2 stage processes proposed by Nelson & Winter (1982, ch. 12)\(^38\).

The equations are as follows:

(a) Productivity associated with an imitiation draw:

1\(^{st}\) stage – choice of best practice (and corresponding productivity \( \pi^M_{i,j,t} \)) to imitate:

\[
\pi^M_{i,j,t} = d_m \max_{i,j} (\pi^0_{i,j,t})
\]

(29).

2\(^{nd}\) stage – probability of imitative success:

\[
Pr(d_m = 1) = 1 - \exp(-\rho_{mi} p_{i,j,t} x_{i,t} a_m)
\]

(30),

where \( d_m \) is a dummy variable representing success (\( d_m=1 \)) or failure (\( d_m=0 \)) of the imitative draw; \( \rho_{mi} \) is the share of revenue spent in imitative R&D; and \( a_m \) is a sector-specific exogenous parameter of “technological opportunity” of imitative success\(^39\).

\(^38\) In the simulations, among the total of eight firms three kinds of firms were assumed to exist: firms numbered 1 and 2 are “strong” innovators (higher R&D innovative spending than imitative spending); those numbered 3 and 5 “weak” innovators (the inverse proportion); and firms 6 through 10 are imitators (only imitative R&D).

\(^39\) To be precise, this is not the only variable reflecting the degree of technological opportunity of a given technology: the exogenous productivity growth of the best practice may be interpreted in a similar way, even moreso since Nelson & Winter. To avoid ambiguity we decided to call the latter effect simply as “productivity gains” of the technological frontier.
(b) Productivity associated with an innovation draw:

1st. stage – probability of innovative success:

\[
\Pr(d_n = 1) = 1 - \exp(-\rho_{ni} p_{it} x_{it} a_n)
\]

(31),

where \(d_n\) is a dummy variable representing success (\(d_n=1\)) or failure (\(d_n=0\)) of the innovative draw; \(\rho_{ni}\) is the share of revenue spent in innovative R&D; and \(a_n\) is a sector-specific exogenous parameter of “technological opportunity” of innovative success.

2nd. stage – productivity obtained by innovation, \(\pi_{it}^N\) (only if \(d_n=1\)):

\[
\log (\pi_{it}^N) \sim N(\mu, \sigma^2)
\]

(32),

where \(\mu\) and \(\sigma\) are given exogenously.

The final choice, that will define the productivity of the firm’s “internal” best practice \(\pi_{it}^F\), will be the technology with the highest productivity among available alternatives:

\[
\pi_{it}^F = \max(\pi_{it-1}^F, \pi_{it}^N, \pi_{it}^M)
\]

(33).

**Block 7: Income generation**

The unit surplus, \(s_t\), can be obtained subtracting from price the indirect taxes and unit costs; however, its total amount can only be defined after sales:

\[
s_t = (1 - \hat{\tau}_t) p_t - \varphi_t
\]

(34),

where \(\hat{\tau}_t\) is the diagonal matrix of indirect tax rates, charged over the sector’s revenue. Total surplus is defined ex post, multiplying its unit value by total sales. The aggregate amount of surplus in the economy, \(TS_t\), will be the sum of all sectors’ surpluses:

\[
TS_t = s_t^T x_t^w
\]

(35).

The total wage, \(TW_t\), analogously, is defined aggregating wages through sectors, including public sector wages \(W_p\). To simplify, a flexible labor contract is implicitly assumed, that is, the amount of labor employed is determined by the level of production and there is no labor supply constraint. Wage unit is subject to change every four periods according to changes in each sector’s average productivity.

\[
w_{jt} = w_{jt-1} \left(1 + \gamma^w \left(\frac{\pi_{jt-1} - \pi_{jt-5}}{\pi_{jt-5}}\right)\right)
\]

(36),

\[
WT_t = w_t x_t + W_p
\]

(37).

The GDP in each production period is given by the sum of total wage and surplus with the indirect taxes:
\[ Y_t = TS_t + TW_t + T^i_t \]  \hspace{1cm} (38).

Finally, the two functional income classes must be converted into \( m \) personal income classes. This is done by a matrix \((m \times 2)\) of personal income appropriation (personal income class x functional income class), as in the equation below:

\[
y^d_i = (I - \hat{\tau}^d)R \begin{bmatrix} DP_t \\ WT_t \end{bmatrix}
\]  \hspace{1cm} (39),

where \( \hat{\tau}^d \) is the \((m \times m)\) diagonal matrix of the income tax rate and \( DP_t \) is total distributed profits.

The real income of each class, \( y^r_{h,t} \), is determined by deflating the corresponding money income by a class-specific consumer price index \((CPI_{h,t})\):

\[
y^r_{h,t} = \frac{y^d_{h,t}}{CPI_{h,t}}
\]  \hspace{1cm} (40), where the index is a Paasche one whose weights are given by the marginal propensity to consume domestic and imported goods by each class.

**Block 8: Public and foreign sectors**

**8A. Public sector**

Government, as mentioned above, is introduced in a partially exogenous and very simplified way. The main components of this block are: government expenses and income and indirect tax revenues. Interest and exchange rate are fixed in this preliminary version, as well as the distribution of the expenses. The latter, based on a surplus target\(^{40}\), are determined every period by the difference between the expected taxes and the target surplus. The former is calculated by past tax revenue, corrected by the expected growth rate. Government expenses, \( g^g_t \), are proportionally divided into wages, consumption and investment:

\[
G_t = g^g_t T_{t-1} - \xi^g Y_{t-1}
\]

\[
C^g_t = G_t c^g \hspace{1cm} W^g_t = G_t w^g \hspace{1cm} I^g_t = G_t \kappa^g
\]  \hspace{1cm} (41);

where \( C^g_t \) and \( I^g_t \) are \((p \times 1)\) vectors; \( c^g, \kappa^g \) and \( w^g \) the proportions.

\(^{40}\) A rule for changing endogenously this target may be subject to simulations.
Government revenue is obtained from indirect and income (direct) taxes. The indirect taxes are paid by sectors according to their sales proceeds. Income taxes are applied over the total amount of personal income of each income class, with class-specific rates.

8B. Foreign sector

The second partially exogenous block is the foreign sector, composed, in this version, exclusively by the trade balance, supposed to be identical to foreign balance of payments\(^{41,42}\).

Exports are determined by a fixed coefficient, \(X_i\), over the “rest of the world” income, \(Y_i\) (measured in domestic currency) and the corresponding income elasticity on the world market, \(\eta_i\). This simple form captures both the general international situation, expressed by the world income, and the sector-specific conditions expressed by the export coefficient and elasticities. In order to define the exports in terms of units of output, this value is divided by each sector’s average price:

\[
e^{x}_{i,t} = X_i \left( \frac{e^{x}_{i,t}}{p_{i,t}} \right)^{\eta_i} \left( Y_i \right)^{\eta_i}
\]

(42).

Aggregate exports are:

\[
X_i = p^{e_i} X_i
\]

(43),

where \(p^{e_i}\) is the \((1 \times p)\) vector of export prices (in domestic currency) and \(e^{x}_i\) the \((p \times 1)\) vector of exports from each sector.

Imports are determined in the same way as the domestic demand. The intermediate goods imports are defined by technical coefficients and planned production; the consumer goods imports are defined by a linear function with increasing lags\(^{43}\) and decreasing marginal propensity; capital goods imports were already explained.

Aggregate imports value in domestic currency is given by the sum total imports by each sector multiplied by the respective international prices and the exchange rate:

\[
M_i = e^{m}_i \left( p^{m}_i \right)
\]

(44),

where \(p^{m}_i\) is the \((1 \times p)\) vector of international prices and \(e^{m}_i\) the \((p \times 1)\) vector of imports of products corresponding to each sector. Finally, it is possible to determine the trade

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\(^{41}\) It does not include compensatory capital flows. Contrary to many models of balance of payments constraint, no assumption is made about the trade balance.

\(^{42}\) The other components of foreign balance of payments will be introduced in a later version of the model.

\(^{43}\) The same lags as in domestic consumption.
balance in domestic currency, which in this preliminary version of the model will be equal to the balance of payments.

4. Preliminary simulation results

The simulation results presented in this section only give a very brief idea of the trajectories generated by the model. Further work will provide a more systematic analysis of the parameters and initial conditions, as well as of the time series generated by the simulations. Also, given the size of the model, a deeper analysis will be made on each block separately. Nevertheless, since the stochastic part of the model is very limited, the results of each simulation run based on the standard or “benchmark” conditions are analytically relevant. Models like this are basically deterministic, although highly path-dependent, being more sensitive to initial conditions than to the random seed of the stochastic component.

In the benchmark setup we assume p=4, n=10 and m=4; the sectors are: consumption (one), intermediate (two) and capital (one). All firms are identical in each sector except for technological and price strategies, according to which they may be divided in three groups: (i) strong innovators – which allocate a larger part of R&D expenses to innovative search and put a higher weight on desired price; (ii) weak innovators –a smaller part of R&D to innovative search, but also a higher weight on desired price; and (iii) pure imitators –all R&D to imitative search and higher weight on average price.

As in the original multisectoral model of Possas (1984) and in the tradition of Kalecki (1954), the main macrodynamic result of the simulations using the benchmark setup, as shown below, is the cyclical behavior of the GDP (fig. 1). Fluctuations are relatively stable, although their pattern, as expected, is more complex than aggregate analytical models (fig. 2). These results are observed in many different simulations, which are not reported here. But two general points deserve attention.

Firstly, just like the traditional neo-Keynesian and Kaleckian models, the regularity of the main observed fluctuations may be explained, in very general lines, by the lagged dual effect of investment, stimulating demand in the short term through multiplier effects and adding productive capacity in a longer term, whose eventual utilization may exceed or lag the desired level, propagating the original impulse. Secondly, a comparative analysis of simulations under different assumptions has shown that the relative stability of the fluctuations, unlike traditional aggregate neo-Keynesian models, is due to a much more complex investment function, where the usually explosive accelerator effect is balanced by the influence of the degree of capacity utilization and by a very effective financial constraint.

Figure 1
Figure 2

Annual Real GDP

Figure 3

Consumption goods sector
A special setting can be used to exhibit an important additional dynamic property of the model, which recalls Kalecki’s (1954) results. The following figures show that, if all technological features of the model are taken out, keeping only the dynamic components related to “effective demand” - i.e. to endogenous interaction between production, sales, consumption and investment -, it is capable of generating fluctuations, but not a long run trend. Of the main possible determinants of long run trend, the model incorporates in this first version exports, autonomous government expenditures and an autonomous investment component. These components, however, are introduced in a way that is unable to generate a significant trend, as opposed to technological autonomous components, which are responsible for a steady positive trend in real income in these simulations, as shown in fig. 5 below.
The co-movements of real GDP, consumption and investment deserve also some comment. As shown in the figures below, although both consumption and investment are pro-cyclical, the latter fluctuates more and the former less than GDP, which is consistent with Keynesian view of investment being more volatile and consumption more stable. This holds irrespectively of technological search being made or not, as can be seen in the figures that follow.
References


